Worksheet 108 Solutions

2000 #5

(a) $y^2 + 2xy\dfrac{dy}{dx} - 3x^2y - x^3\dfrac{dy}{dx} = 0$

$$\frac{dy}{dx}\left(2xy - x^3\right) = 3x^2y - y^2$$

$$\frac{dy}{dx} = \frac{3x^2y - y^2}{2xy - x^3}$$

$$2\begin{cases} 1: \text{implicit differentiation} \\ 1: \text{verifies expression for } \dfrac{dy}{dx} \end{cases}$$

(b) When $x = 1$, $y^2 - y = 6$

$$y^2 - y - 6 = 0$$

$$(y-3)(y+2) = 0$$

$$y = 3,\ y = -2$$

At $(1,3)$, $\dfrac{dy}{dx} = \dfrac{9-9}{6-1} = 0$

Tangent line equation is $y = 3$

At $(1,-2)$, $\dfrac{dy}{dx} = \dfrac{-6-4}{-4-1} = \dfrac{-10}{-5} = 2$

Tangent line equation is $y + 2 = 2(x-1)$

$$4\begin{cases} 1: & y^2 - y = 6 \\ 1: & \text{solves for } y \\ 2: & \text{tangent lines} \end{cases}$$

Note: 0/4 if not solving an equation of the form $y^2 - y = k$

(c) Tangent line is vertical when $2xy - x^3 = 0$

$x\left(2y - x^2\right) = 0$ gives $x = 0$ or $y = \dfrac{1}{2}x^2$

There is no point on the curve with x-coordinate 0.

When $y = \dfrac{1}{2}x^2$, $\dfrac{1}{4}x^5 - \dfrac{1}{2}x^5 = 6$

$$-\frac{1}{4}x^5 = 6$$

$$x = \sqrt[5]{-24}$$

$$3\begin{cases} 1: & \text{sets denominator of } \dfrac{dy}{dx} \text{ equal to } 0 \\ 1: & \text{substitutes } y = \dfrac{1}{2}x^2 \text{ or } x = \pm\sqrt{2y} \\ & \text{into the equation for the curve} \\ 1: & \text{solves for } x\text{-coordinate} \end{cases}$$

2005 (Form B) #5

(a) $2yy' = y + xy'$ $(2y - x)y' = y$ $y' = \dfrac{y}{2y - x}$	$2: \begin{cases} 1: \text{implicit differentiation} \\ 1: \text{solves for } y' \end{cases}$
(b) $\dfrac{y}{2y - x} = \dfrac{1}{2}$ $2y = 2y - x$ $x = 0$ $y = \pm\sqrt{2}$ $(0, \sqrt{2}), (0, -\sqrt{2})$	$2: \begin{cases} 1: \dfrac{y}{2y - x} = \dfrac{1}{2} \\ 1: \text{answer} \end{cases}$
(c) $\dfrac{y}{2y - x} = 0$ $y = 0$ The curve has no horizontal tangent since $0^2 \neq 2 + x \cdot 0$ for any x.	$2: \begin{cases} 1: y = 0 \\ 1: \text{explanation} \end{cases}$
(d) When $y = 3$, $3^2 = 2 + 3x$ so $x = \dfrac{7}{3}$. $\dfrac{dy}{dt} = \dfrac{dy}{dx} \cdot \dfrac{dx}{dt} = \dfrac{y}{2y - x} \cdot \dfrac{dx}{dt}$ At $t = 5$, $6 = \dfrac{3}{6 - \frac{7}{3}} \cdot \dfrac{dx}{dt} = \dfrac{9}{11} \cdot \dfrac{dx}{dt}$ $\left.\dfrac{dx}{dt}\right\vert_{t=5} = \dfrac{22}{3}$	$3: \begin{cases} 1: \text{solves for } x \\ 1: \text{chain rule} \\ 1: \text{answer} \end{cases}$